OCR
Oxford Cambridge and RSA

Friday 16 June 2017 – Afternoon

AS GCE MATHEMATICS

4728/01 Mechanics 1

QUESTION PAPER

Candidates answer on the Printed Answer Book.

OCR supplied materials:
- Printed Answer Book 4728/01
- List of Formulae (MF1)

Other materials required:
- Scientific or graphical calculator

Duration: 1 hour 30 minutes

INSTRUCTIONS TO CANDIDATES

These instructions are the same on the Printed Answer Book and the Question Paper.

- The Question Paper will be found inside the Printed Answer Book.
- Write your name, centre number and candidate number in the spaces provided on the Printed Answer Book. Please write clearly and in capital letters.
- **Write your answer to each question in the space provided in the Printed Answer Book.** If additional space is required, you should use the lined page(s) at the end of the Printed Answer Book. The question number(s) must be clearly shown.
- Use black ink. HB pencil may be used for graphs and diagrams only.
- Answer **all** the questions.
- Read each question carefully. Make sure you know what you have to do before starting your answer.
- Do **not** write in the barcodes.
- You are permitted to use a scientific or graphical calculator in this paper.
- Give non-exact numerical answers correct to 3 significant figures unless a different degree of accuracy is specified in the question or is clearly appropriate.
- The acceleration due to gravity is denoted by g m s^{-2}. Unless otherwise instructed, when a numerical value is needed, use $g = 9.8$.

INFORMATION FOR CANDIDATES

This information is the same on the Printed Answer Book and the Question Paper.

- The number of marks is given in brackets **[ ]** at the end of each question or part question on the Question Paper.
- **You are reminded of the need for clear presentation in your answers.**
- The total number of marks for this paper is **72**.
- The Printed Answer Book consists of **12** pages. The Question Paper consists of **4** pages. Any blank pages are indicated.

INSTRUCTION TO EXAMS OFFICER/INVIGILATOR

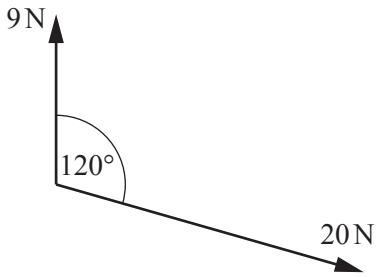
- Do not send this Question Paper for marking; it should be retained in the centre or recycled. Please contact OCR Copyright should you wish to re-use this document.

DC (LK/SG) 137673/2

Answer **all** the questions.

1 AB is a line of greatest slope on a smooth plane inclined at 60° to the horizontal, with A above the level of B. A particle is projected down the plane with speed $3\,\text{m s}^{-1}$ from A towards B. Given that 0.7 s after the instant of projection the particle passes through B, calculate the distance AB and the speed of the particle when it passes through B. **[5]**

2



Two horizontal forces of magnitudes 9 N and 20 N act along bearings 000° and 120° respectively. Calculate the magnitude and the bearing of their resultant. **[6]**

3 Two particles A and B are moving in the same direction along the same straight line on a smooth horizontal surface. A has mass 0.2 kg and velocity $6\,\text{m s}^{-1}$. B has mass 0.3 kg and velocity $4.2\,\text{m s}^{-1}$. The particles collide, and 2 seconds after they collide the distance AB is 1.6 m.

(i) Calculate the velocities of both particles after the collision. **[6]**

(ii) Find the change in the momentum of A as a result of the collision. **[2]**

4 A small ball is projected vertically upwards with speed $18\,\text{m s}^{-1}$ from a point O on the ground. At the same instant a small object is released from rest at a point 27 m vertically above O.

(i) Verify that the ball and the object collide 1.5 s after they are set in motion. **[4]**

(ii) Find the velocities of the ball and the object immediately before they collide. **[3]**

The ball and the object have equal mass. When the ball and the object collide, they coalesce.

(iii) Find the time after their collision when they strike the ground at O. **[5]**

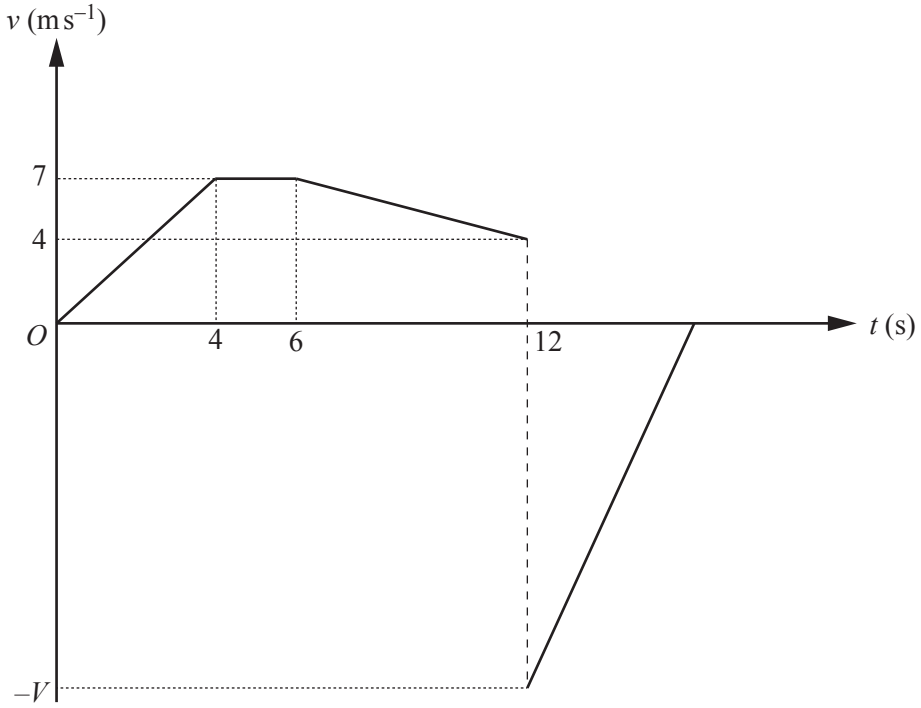
5 A particle moves in a straight line on a horizontal surface. At time t s after being released from rest at a point O on the line, the particle has a velocity $v\,\text{m s}^{-1}$ and a displacement from O of x m. It is given that

$$v = 0.8t^3 - 4t^2 + 5.6t.$$

(i) Find the positive values of t at which the particle has its maximum and minimum velocities, and calculate the values of these velocities. **[5]**

(ii) Express x in terms of t, and hence find the distance travelled by the particle while it is decelerating. **[6]**

6



A particle P is released from rest at a point A on an inclined plane with a variable coefficient of friction. P descends along a line of greatest slope $ABCD$. The velocity of P is $v\,\mathrm{m\,s^{-1}}$ at time $t\,\mathrm{s}$ after its release. The diagram shows the (t, v) graph for the motion of P. When $t = 4$, the particle passes through B with $v = 7$.

(i) Find the constant acceleration of P during the first 4 seconds of its motion. **[1]**

P moves from B to C with constant velocity. P passes through C when $t = 6$. The particle P has mass $0.2\,\mathrm{kg}$ and the frictional force acting on P between B and C has magnitude $0.4\,\mathrm{N}$.

(ii) Find the inclination of the plane to the horizontal, and the magnitude of the normal component of the contact force exerted on P by the plane. **[4]**

P moves from C to D with constant deceleration. P reaches D when $t = 12$ with $v = 4$.

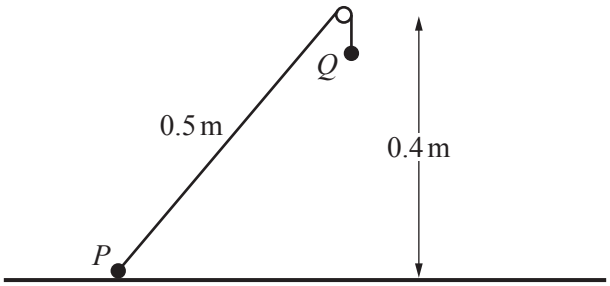
(iii) Show that the frictional force acting on P between C and D has magnitude $0.5\,\mathrm{N}$. **[3]**

Immediately after reaching D at $t = 12$, the particle P is projected with speed $V\,\mathrm{m\,s^{-1}}$ from D back up the line of greatest slope, and comes to rest at C.

(iv) Find V. **[5]**

Question 7 begins on page 4.

7



A particle P of mass 0.4 kg is attached to one end of a light inextensible string. The string passes over a small smooth fixed pulley, and a particle Q of mass 0.1 kg is attached to the other end of the string. P rests in limiting equilibrium on a horizontal surface which is 0.4 m below the pulley, with the string taut and in the same vertical plane as P, Q and the pulley. P is 0.5 m from the pulley (see diagram).

(i) Calculate the coefficient of friction and the magnitude of the contact force exerted on P by the surface. **[7]**

Q is now moved to the position on the surface below the pulley such that the portion of the string attached to Q is vertical. P hangs freely below the pulley and the portion of the string attached to P is vertical. Both particles are at rest when Q is released.

(ii) Find the acceleration of the particles and the tension in the string while P is descending. **[5]**

P strikes the surface and remains at rest. Q comes to instantaneous rest immediately before reaching the pulley.

(iii) Find the length of the string. **[5]**

END OF QUESTION PAPER

Answer			Marks	Guidance	
1		$a = g$ sin60 (= 8.487..) AB = 3x0.7 + $g$sin60 x0.7^2 /2 AB = 4.18 m v = 3 + 9.8sin60 x 0.7 v = 8.94 m s^{-1} *OR* $v^2 = 3^2 + 2xg$sin60 x 4.179 v = 8.94 m s^{-1}	B1 M1 A1 M1 A1 [5] M1 A1	Need not be evaluated SorC AB=(3+8.94)x0.7 /2 may be done second SorC SorC	Both method marks given if $a = g$ v = 3+$g$sin60 x 0.7 may be done first to find v = 8.94, (M1A1) then $8.94^2 = 3^2 + 2g.AB$ to give AB = 4.18 m (M1A1)
2		Applies cosine rule $R^2 = 9^2 + 20^2 - 2x9x20\cos60$ R = 17.3N Applies sine rule sinθ /9 = sin60 /17.3 = sinα/20 Bearing = (0)93.3° *OR* cmpt(@090° = 20cos30 (=10√3) and cmpt@180° = 20sin30 – 9 (=1) $R^2 = (10\sqrt{3})^2 + 1^2$ R = 17.3N tanθ = 1/ 10√3 or tanα= 10√3 /1 Bearing = (0)93.3	M1 A1 A1 M1 A1√ A1 [6] M1 A1 A1 M1A1√ A1	Sides 9,20 and angle 30 or 60 or 120 √301 is OK ft cv(R, angle) From θ = 26.7°or α = 86.7° CorS and SorC Finding two perpendicular components √301 ft cv(cmpts,R) From θ = 3.3	17.3 N is also the eastwards component of the resultant, so there must be evidence that an answer of 17.3 N has been correctly obtained. Accept 093, but not 93 *OR* other pair of perpendicular cmpts 9cos60+20cos60 = 14.5 20sin60 – 9sin60 = 9.5262.. from resolving // and perp angle bisector Accept 093, but not 93
3	**i**	Before Mom. = 0.2x6 + 0.3x4.2 "After" speed difference = 1.6/2 (=0.8) Uses momentum conservation 0.2x6 + 0.3x4.2 = 0.2v + 0.3x(v+0.8) A's speed = 4.44 m s^{-1} B's speed = 5.24 m s^{-1}	B1 B1 M1 A1 A1 A1 [6]	2.46 *OR* 2b – 2a = 1.6 Need not use speed difference If the motion of A is reversed then the RHS starts -0.2v	**Allow 0.2x6 + 0.3x4.2 = 0.2a + 0.3b which is awarded M1A1 if after speeds different**
	ii	Momentum change = 0.2x(6-4.44) Change = 0.312 kg m s^{-1}	M1 A1 [2]	*OR* 0.3x(5.24-4.2) = 0.312	After speed is numerical, non-zero; signs consistent with ans(i) Disregard sign/increase/decrease

4	**i**	Object fall = $9.8x1.5^2/2$ Ball rise = $18x1.5 - 9.8x1.5^2/2$ Distance = 11.025 + 15.975 Distance = 27 m AG *OR* Distance fallen = $9.8t^2/2$ Distance risen = $18t - 9.8t^2/2$ $9.8t^2/2 + 18t - 9.8t^2/2 = 27$ $t = 1.5$ AG	B1 B1 M1 A1 [4] B1 B1 M1 A1	11.025 m 15.975 m Appropriate signs and full accuracy Solves total of distances equation	**SC** 18x1.5 = 27 B1 only but $9.8x1.5^2/2 + 18x1.5 - 9.8x1.5^2/2 = 27$ B1B1M1A1 $9.8t^2/2$ without the context of "distance fallen" is B0. Similarly for $18t - 9.8t^2/2$
	ii	Object vel (=9.8x1.5) = 14.7 m s^{-1} (down) Ball vel =+/-(18-9.8x1.5) Ball vel = 3.3 (upwards)	B1 M1 A1 [3]	Accept -14.7 m s^{-1} Must be a difference expression 3.3 if v=18 or -3.3 if v = -18	Candidates may find object velocity and ball velocity in (i). **These answers must be quoted here for 3 marks to be given.**
	iii	$14.7m - 3.3m = 2mu$ $u = 5.7$ $15.975 = 5.7t + 9.8t^2/2$ Solve $4.9t^2 + 5.7t - 15.975 = 0$ $t = 1.32$ s	M1 A1 M1* D*M1 A1 [5]	Momentum conservation; after mass = 2xbefore mass Must use coalesced velocity and s<27 3 term QE and evidence of method of solution if answer incorrect.	Disregard signs $v^2 = 5.7^2 + 2x9.8x15.975$ and $v = 5.7 + 9.8t$ Create both M1* Find v (=18.537..), solve for t D*M1 Answer t = 1.32 s
5	**i**	Differentiates to find accn $dv/dt = 3x0.8t^2 - 2x4t + 5.6$ Solve $2.4t^2 - 8t + 5.6 = 0$ t = 1, 2.33(33..) (Accept7/3) v = 2.4 m s^{-1}, 1.45 m s^{-1}	M1* A1 D*M1 A1 A1 [5]	 3 term QE and evidence of method of solution if answer incorrect. *OR* t=1 and v=2.4 m s^{-1} A1 *OR* t=7/3 and v=1.45 m s^{-1} A1	 As there are two values needed for each A1 mark, accept values correct to 2 sig fig.
	ii	$x = \int 0.8t^3 - 4t^2 + 5.6t\,dt$ $x = 0.8t^4/4 - 4t^3/3 + 5.6t^2/2$ (+c) $x = 0.2t^4 - 4t^3/3 + 2.8t^2$ $x(2.3333) - x(1) =$ $(0.2x2.3333^4 - 4x2.3333^3/3 + 2.8x2.3333^2)$ $- (0.2x1^4 - 4x1^3/3 + 2.8x1^2)$ Distance = 2.57 m	M1 A1 A1 M1* D*M1 A1 [6]	 $x = 0.2t^4 - 1.33t^3 + 2.8t^2$ Evaluates x at two times found from a = 0 Subtraction of values (4.23-1.67)	 Simplified coefficients and c discarded These are the answers in (i)

6	**i**	a = 1.75 m s^{-2}	B1 [1]	Accept 7/4 as the final answer or 1 and 3/4	
	ii	$0.2g\sin\theta$ -0.4 = 0 θ = 11.8° $R = 0.2g\cos 11.776$ R = 1.92 N *OR* $R^2 = (0.2g)^2 - 0.4^2$ R = 1.92 N	M1 A1 M1 A1 [4] M1 A1	SorC Difference of two forces 11.776..° CorS Angle is numerical 1.9187 N	
	iii	Deceleration = +/-(4-7)/(12-6) = +/-0.5 N2L with numerical accn and wt cmpt $0.2(4-7)/(12-6) = 0.2g\sin 11.8 - F$ F = 0.5 N AG	M1 M1 A1 [3]	 Accept 0.4 as wt cmpt $0.2(4-7)/(12-6) = 0.4 - F$	Ignore signs
	iv	DC = (4+7)x(12-6) /2 DC = 33 m $0.2a$ = +/-(0.5+0.4) $0 = V^2$ -2x4.5x33 V = 17.2	M1 A1 M1* D*M1 A1 [5]	This calculation uses values from the third portion of the graph N2L with two force terms (a = +/-4.5) M0 if $a = g$	Award these marks even if subsequent work also incorporates other portions of the graph *ABCD* (=14+14+33) = 61.

7	**i**	$\sin\theta = 0.4/0.5$ or $\cos\theta = 0.3/0.5$ $T = 0.1g$ (=0.98) N $Fr = T\cos\theta$ (= 0.588) $R = 0.4g - T\sin\theta$ (= 3.136) μ (= 0.588 /3.136) = 3/16 or 0.1875 $C^2 = 0.588^2 + 3.136^2$ C = 3.19 N	B1 B1 M1 M1 A1 M1 A1 [7]	θ is angle between string and horizontal CorS. T, angle do not have to be numerical SorC. T, angle do not have to be numerical with $0.4g$ 0.187 or 0.188 Must have two non-zero numerical values	If two values of T are employed, award B1 for $0.1g$ associated with Q. R must be a difference of forces
	ii	$0.4g-T = 0.4a$ $T-0.1g = 0.1a$ $0.3g = 0.5a$ *OR* 0.4g - 0.1g = 0.4a + 0.1a a = 5.88 m s^{-2} T = 1.568 N = 1.57 N	M1 A1 M1 A1 A1 [5]	N2L for either particle, no components Both equations correct Solves two simultaneous equations	Finding a correctly from the combined equation gets M1A1. Using a in an N2L equation for P or Q can get M1, and obtaining the correct value of T gets A1, hence 4 marks out of 5
	iii	P descends = x m (= (2x0.4 - l) m) $v^2 = 2x5.88x$ (=11.76x) $0 = v^2 - 2g(0.4-x)$ x = 0.25 String is 0.8-0.25 m long l = 0.55 m *OR* (P starts d m below pulley) $v^2 = 2x5.88(0.4 - d)$ $v^2 = 2gd$ d = 0.15 String is 0.4+0.15 m long l = 0.55 m	 M1 M1 A1 M1 A1 [5] M1 M1 A1 M1 A1	 P and Q moving together Q rising alone P and Q moving together Q rising alone	 Eqn has two unknowns Eqn has two unknowns Eqn has two unknowns Eqn has two unknowns